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Topic 8

Capacitor Circuits

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This lecture will be about an additional component – the capacitor. Unlike resistor, the behaviour of the current flowing through a capacitor and the voltage across a capacitor depends on whether the signal is a dc voltage source, an ac voltage source (e.g. a sine wave) or a step signal (such as a square or clock signal).

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Capacitors & Capacitance

- A capacitor is formed from two conducting plates separated by a thin insulating layer called a **dielectric**.

- If a current i flows, positive change, q, will accumulate on the upper plate. To preserve charge neutrality, a balancing negative charge will be present on the lower plate.
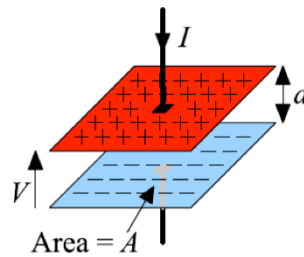
- There will be a potential energy difference (or voltage v) between the plates proportional to q.

$$v = \frac{d}{A\epsilon}q$$

where A is the area of the plates, d is their separation and ε is the permittivity of the insulating layer ($\varepsilon_0 = 8.85\ pF/m$ for a vacuum).

- The quantity $C = A\varepsilon/d$ is the capacitance and is measured in Farads (F). Hence $q = Cv$, and the current i is the rate of charge on the plate.

The capacitor equations:

$$q = Cv, \quad therefore \quad \frac{dq}{dt} = i = C\frac{dv}{dt} \quad and \quad v = \frac{1}{C}\int i\,dt$$

P326-327

Besides resistors, capacitors are one of the most common electronic components that you will encounter. Sometimes capacitors are components that one would deliberately add to a circuit. Other times, capacitors are side effects that come about even if we don't want them.

The simplest capacitor is formed by an insulating material (known as dielectric) sandwiched between two parallel conducting plates. When a voltage potential is applied to the two ends, charge accumulates on the plates.
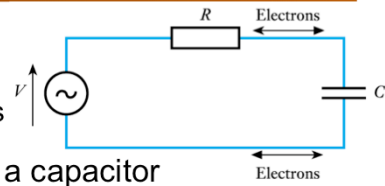
In capacitors, voltage v is proportional to the charged stored q. The constant of proportionality is the capacitance C. Since current i is the rate of change of charge (i.e. the flow of charge), the relationship between v and i involves differentiation or integration.

Capacitance is measured in Farads. However, a 1F capacitor is consider large! Most of the capacitors you will use in this module are of the order of microFarad (uF).

DC, AC and Capacitors

- A constant current (DC) cannot flow through a capacitor
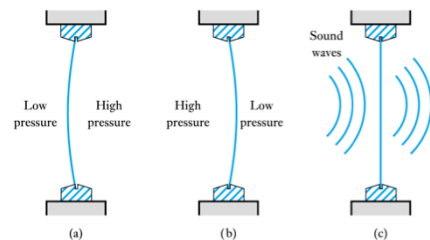 - There is an insulator between the two terminals
- An alternating current (AC) can "flow through" a capacitor
  - Since the voltage across a capacitor is proportional to the charge on it, an alternating voltage must correspond to an alternating charge
  - This can give the impression that an alternating current flows through the capacitor
- A mechanical analogy:
  - Air (charge) cannot pass through a window in spite of the pressure difference (voltage potential)
 - However, alternating pressure can make the window vibrates, produces air movement

Since there is an insulating layer between the two conducting plates of a capacitor, DC current cannot flow through a capacitor. So always remember: **A CAPACITOR IN SERIES BLOCKS DC part of a signal**. However, alternating or changing current can flow through a capacitor. The best analogy is the flow of air from inside to outside of the building. Assuming that the window is completely sealed, air inside the building cannot flow to the outside in spite of the pressure difference between the two sides. The pressure difference is analogous to the voltage potential at the two end of the capacitor. The air flow is like DC current.

However, if the air pressure difference is alternating, there can be air movement on both sides as shown in the diagram.

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Capacitors in Series and in Parallel

◆ **Capacitors in parallel**

- consider a voltage V applied across two capacitors
- then the charge on each is
  $Q_1 = VC_1$ and $Q_2 = VC_2$
- if the two capacitors are replaced with a single capacitor C which has a similar effect as the pair, then

  Charge stored on combined C is $Q = Q_1 + Q_2$
  $$\Rightarrow VC = VC_1 + VC_2$$
  $$\Rightarrow C = C_1 + C_2$$
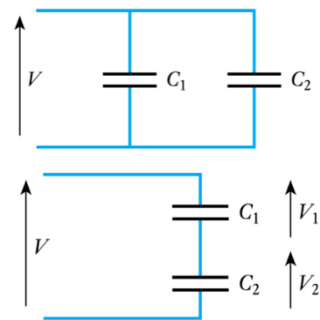
◆ **Capacitors in series**

- consider a voltage V applied across two capacitors in series
- the only charge that can be applied to the lower plate of C_1 is that supplied by the upper plate of C_2. Therefore the charge on each capacitor must be identical.
- Let this be Q, and therefore if a single capacitor C has the same effect as the pair, then:

  $V = V_1 + V_2 \Rightarrow Q/C = Q/C_1 + Q/C_2$
  $$\Rightarrow 1/C = 1/C_1 + 1/C_2$$

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Connecting two capacitors in parallel results in their capacitances ADD-ed together (just like resistors in series).

Connecting two capacitors in series results in their capacitances combined in a product/sum manner, similar to two parallel resistances.

The Exponential Signal

$$y(t) = A(1 - e^{-t/\tau})$$

$$y(t) = Ae^{-t/\tau}$$
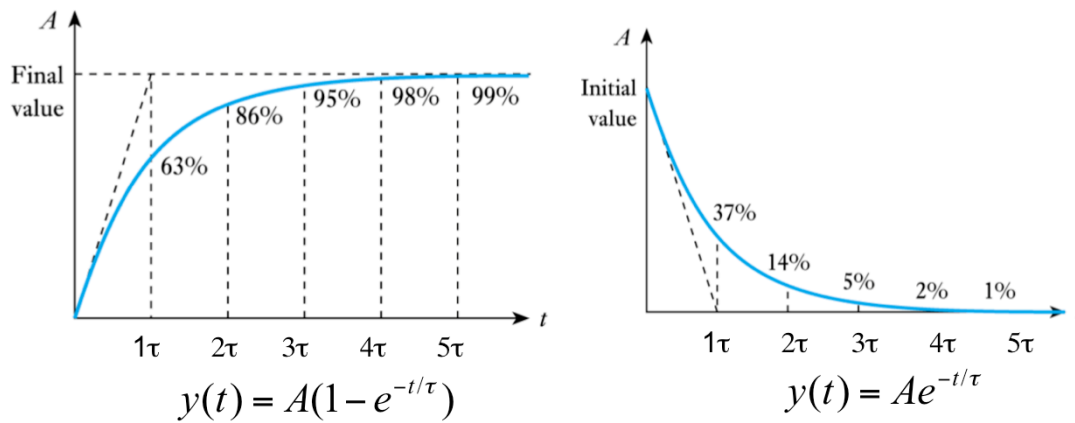
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Before we embark on circuits using capacitors, let us examine one of the signals that you explored in Lab 1 in the past week – the exponential signal.

Exponential signals are interesting. Here the rate of change is shown in terms of the **time constant** τ (tau).

The following facts are worth remembering:

1. For exponential rise, the signal reaches 63% at one τ, and 95% at 3τ.
2. For exponential fall, the signal reaches 37% at one τ, and 5% at 3τ.

# Capacitor and the Exponential

Consider <u>the</u> circuit shown here:

- ◆ Initially, the switch is in the down position. The input is connected to GND and the capacitor is discharged.
- ◆ At t = 0, the switch goes to up position. The battery voltage Vs is applied to the RC circuit and the capacitor starts charging.

- ◆ At t = 0, V(t) is initially zero. The voltage across R is initially Vs. Therefore the charging current is Vs/R.

- ◆ As the capacitor charges:
  - ➤ *V(t)* increases
  - ➤ V_R (voltage cross the resistor) decreases
  - ➤ *I(t)* the charging current decreases
  - ➤ This result in the exponential behaviour of both *V(t)* and *I(t)*


Charging a Capacitor
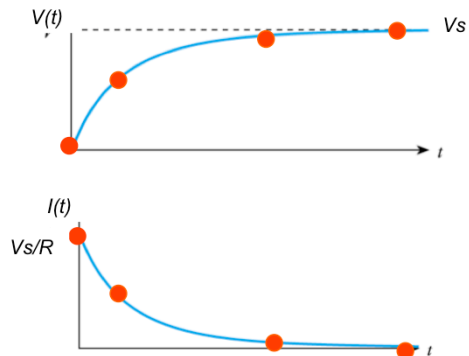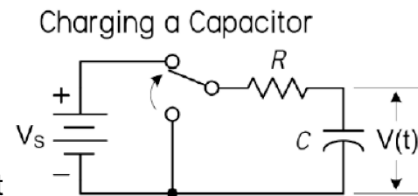
For the circuit shown here, assume the capacitor has zero charge (and 0v) at t = 0. The switch is closed, connecting the circuit to the constant voltage source Vs. Initially the voltage drop across the resistor is Vs. A current of Vs/R flows from the source to capacitor. However, as V increases, the current I decreases. This results in the exponential drop of changing current and an exponential rise of the capacitor voltage. We will examine mathematically how i(t) and V(t) changes over time later.

RC circuit and time constant

Charging a Capacitor
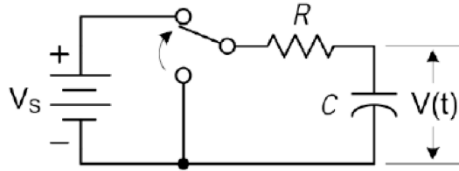
$$V(t) = V_S(1 - e^{-t/RC})$$
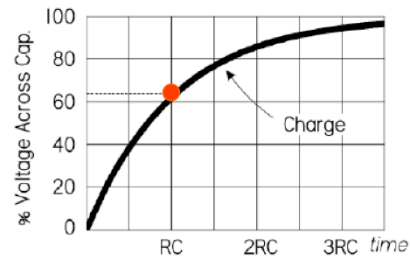
◆ **Time constant**
   • Charging current I is determined by R and the voltage across it
   • Increasing R will increase the time taken to charge C
   • Increasing C will also increase time taken to charge C
   • Time required to charge to a particular voltage is determined by **CR**
   • This product **CR** is the **time constant** τ (greek tau)

P327-328

For now, it is important to consider the parameter known as the time constant. If R is large, the charging current I is small, and it takes longer to charge the capacitor. For a given R, if C is large, it can store more charge for a given voltage, therefore the time needed to charge a capacitor to a certain voltage is proportional to the produce R x C. RC is known as the time constant of this circuit.

◆ Consider what happens to the circuit shown here as the switch is closed at t = 0.

◆ Apply KVL around the loop, we get:

$$iR + v = Vs, \quad but \quad i = C\frac{dv}{dt} \quad therefore \quad RC\frac{dv}{dt} + v = Vs$$
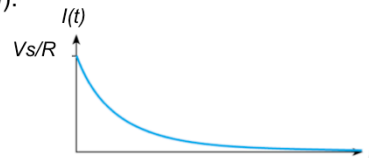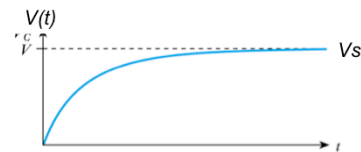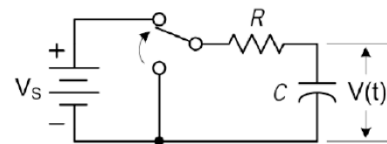
Charging a Capacitor

◆ This is a simple **first-order differential equation** with constant coefficients.

◆ Assuming $V(t) = 0$ at $t = 0$, the solution to this is:

$$V(t) = V_S(1- e^{-t/RC})$$

◆ Since $i = C\frac{dv}{dt}$ this gives (assuming V(t) = 0 at t = 0):

$$i = I \times e^{-\frac{t}{RC}} = I \times e^{-\frac{t}{\tau}}, \quad where \quad I = \frac{Vs}{R}$$

We will now derive the exponential equation formally.

For that you need to be familiar with solving first-order differential equations from your maths lectures.

We want to solve: $\quad RC\frac{dv}{dt} + v = Vs$

$$\frac{dv}{dt} = \frac{Vs - v}{RC}$$

$$\Rightarrow \frac{dt}{RC} = \frac{dv}{Vs - v}$$

Integrate both sides, we get:

$$\frac{t}{RC} = -\ln(Vs - v) + A, \quad where \ A \ constant \ of \ integration$$

Use boundary condition: when t= 0, v = 0:
$$\frac{0}{RC} = -\ln(Vs - 0) + A$$
$$\Rightarrow A = \ln Vs$$

Therefore

$$\frac{t}{RC} = -\ln(Vs - v) + \ln Vs = \ln\frac{Vs}{Vs - v}$$

$$\Rightarrow e^{\frac{t}{RC}} = \frac{Vs}{Vs - v} \Rightarrow v = Vs(1 - e^{-\frac{t}{RC}})$$
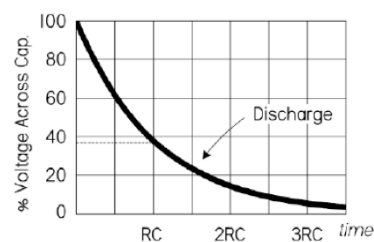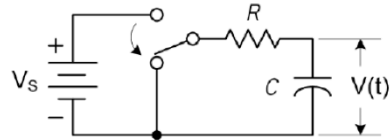
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Discharging Capacitor in a RC circuit

- Consider what happens to the circuit shown here as the left switch is open and the right switch closed at t = 0.
- At $t = 0$, $V(t)$ = Vs.
- Apply KVL around the right loop, we get:

Discharging a Capacitor

$$iR + v = 0, \quad and \quad i = C\frac{dv}{dt} \quad therefore \quad RC\frac{dv}{dt} + v = 0$$

- Solving this simple first-order differential equation gives:

$$V(t) = V_S \, e^{-t/RC}$$

Let us now consider what happens if we charge up the capacitor, then at t = 0, discharges it. The equations is also easy to solve and it is clear that the discharge profiles in V and I also follow the exponential curves.

DC Blocking using Capacitor

- ◆ Capacitor is often used to present dc voltage from passing from one side of the circuit to another.
- ◆ Here, on the left side, the signal has a 3V DC component, and a sinewave superimpose.
- ◆ On the right side, the output signal Vout is centred around 0V. That is, the DC input is "blocked" or isolated from the output.
- ◆ This use of capacitor is also known as "**AC coupling**".
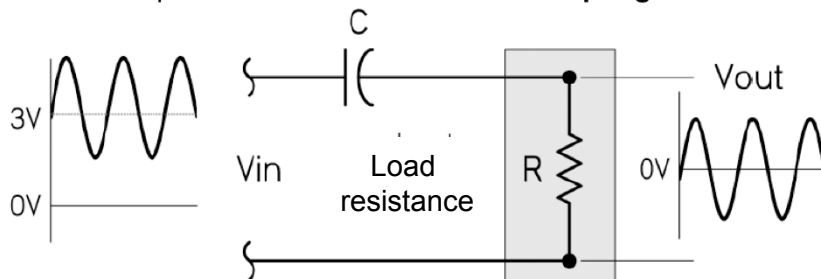
P343

We will now consider some common use of capacitors in electronic circuit. The first application of a capacitor is to remove or "block" DC part of an electrical signal.

Consider the circuit above. The input signal Vin has a 3V DC voltage on which is a sinewave. By choosing the correct value for C and R, you can obtain an output signal, Vout, which is has 0V DC (i.e. DC is blocked) and only the sinusoidal signal remains.

The capacitor is acting like a window of an airplane. The constant pressure on outside the airplane (DC value) is not affecting the pressure inside the cabin. However, vibration of the window (sinusoidal component) can pass through to the cabin if the window is sufficient flexible relative to the "resistance" of the cabin air.

Filtering effect of Capacitor

- Such circuit also has different effect on the input signal at different frequencies.
- Shown here is two signals, one at 5Hz and another at 100Hz and the C and R values are as given.
- The 5Hz sinewave is suppressed by -30dB or reduced by a factor of 32.
- The 100Hz signal is only reduced by -5.48dB or reduced by a factor of 1.9.
- Therefore, a C in series with a R as shown will give us a high pass filter: a circuit that passes high frequency signals but suppresses low frequency.
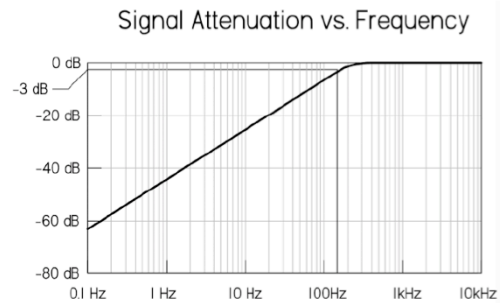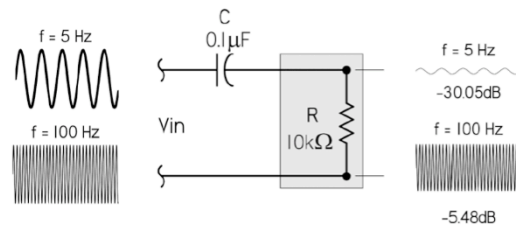
Let us take another view of this circuit besides its DC blocking (or AC coupling) quality.

If C= 0.1uF and R = 10kΩ, you will find that the circuit will strongly suppress a 5Hz sinewave, but if the input signal has a much higher frequency (say 100 Hz), the signal is passed through the capacitor with little or no reduction.

If you plot signal gain (or attenuation), i.e. Vout/Vin, vs Frequency, you can a characteristic as shown in the slide. The low the Y-axis value, the lower the gain and stronger the suppression. In fact if you project the plot toward frequency = 0 (that is, to DC), the gain is −infinity (dB). That's why this circuit can "BLOCK" DC signal.

Note that the plot here has both axes in logarithmic scale. The frequency axis is clearly log scale. How about the y-axis? It is plotting the gain of the circuit in dB or decibel. dB is also in log scale as will be shown in the next slide.

Decibel (dB)

- Ratio of output to input voltage in an electronic system is called voltage **gain**:

$$A = \left| \frac{V_{out}}{V_{in}} \right|$$

- If the gain is low than 1, we also call this attenuation.
- Voltage gain of a circuit is often expressed in logarithmic form:

$$A\ (in\ dB) = 20\ \log_{10} \left| \frac{V_{out}}{V_{in}} \right|$$

- Power gain of a circuit is the ratio of output power to input power, and is also often expressed in dB, but the equation is different:

$$Power\_Gain\ (in\ dB) = 10\ \log_{10} \left| \frac{P_{out}}{P_{in}} \right|$$

P204-205

In electronics, we often ask the question: what is the ratio of the output to input signal? This ratio is important. If it is larger than 1, the electronic circuit provides gain (we also call this amplification as will be seen in a later lecture).

If the ratio is lower than 1, then the circuit provides attenuation (or suppression).

However, we often express this ratio or gain not just as such a ratio, but in logarithmic scale. Why? It turns out that expressing such ratio in log scale provide us with much higher **dynamic range**. For example, our human perception is generally in log scale, not linear scale. Our hearing and seeing sensitivity is not linear, but logarithmic.

Log - base 10 of the ratio is known as a bel. Scaling this further up by a factor of 10 is known as decibel. That ratio is generally considering the ratio of output power to input power (not voltage). However, since power is voltage square, we found that the common equation to find voltage gain (ratio) in decibel is given by the equation:

$$A\ (in\ dB) = 20\ \log_{10} \left| \frac{V_{out}}{V_{in}} \right|$$

This is an equation that you must commit to memory – very useful for many things!

Types of Capacitors

- Capacitor symbol represents the two separated plates. Capacitor types are distinguished by the material used as the insulator.

- **Polystyrene**: Two sheets of foil separated by a thin plastic film and rolled up to save space. Values: 10 pF to 1 nF.

- **Ceramic**: Alternate layers of metal and ceramic (a few µm thick). Values: 1 nF to 1 µF.

- **Electrolytic**: Two sheets of aluminium foil separated by paper soaked in conducting electrolyte. The insulator is a thin oxide layer on one of the foils. Values: 1 µF to 10mF.

- Electrolytic capacitors are **polarised**: the foil with the oxide layer must always be at a positive **voltage** relative to the other (else **explosion**).

- Negative terminal indicated by a curved plate in symbol or "-".
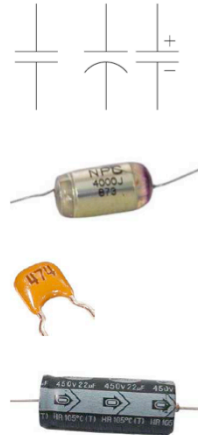
P333-340

There are many different types of capacitors depending on the method of construction and the materials used. The most common three types are: polystyrene, ceramic and electrolytic. You will choose which to use depending on the operating signal frequency. Polystyrene and ceramic capacitors are good for a wide frequency range, particularly at very high frequencies (say over 1MHz). However, they only come in fairly low capacitance value.

For large capacitances, one would use electrolytic capacitors. Electrolytic capacitors are only good for fairly low frequencies. Furthermore, they have polarity, i.e. it has a positive and a negative terminal. You must connect the capacitor +ve terminal to the more positive voltage than the –ve terminal.
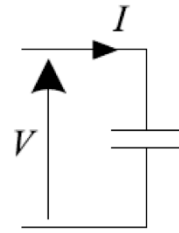
Current / Voltage Continuity

Capacitor: $i = C \, dv / dt$

◆ For the voltage to change abruptly $dv / dt = \infty \Rightarrow i = \infty$.

This never happens so ...

◆ **The voltage across a capacitor never changes instantaneously.**

◆ Informal version: A capacitor "tries" to keep its voltage constant.

I

V

The take-home message that you must remember is that:

Capacitor tries to keep its voltage constant.

Summary

◆ Capacitor:

- $i = C\,dv\,/dt$
- parallel capacitors add in value
- v across a capacitor never changes instantaneously
- When charging a capacitor with a constant DC voltage through a resistor, the capacitor voltage rises exponentially
- The time constant of the exponential is the product of R and C.